DEPARTMENT OF PHYSICS

AUL DEGREE COLLEGE, AUL

<u>QUESTION BANK</u>

+3 1ST SEMESTER PHYSICS HONS. (NEP 2020 SYLLABUS)

- **MAJOR PAPER-1 (CORE-1): MATHEMATICAL PHYSICS-I**

- **MAJOR PAPER-2 (CORE-2): MECHANICS**

COMPREHENSIVE QUESTION BANK

Course: Mathematical Physics-I (Semester I)

As per Choice Based Credit System (CBCS) Syllabus & Course Outcomes

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Course Outcomes & Syllabus Overview

Outcomes: Basic understanding of differential equations, vector calculus (differentiation & integration), Dirac delta function, orthogonal curvilinear coordinates, and error analysis.

Unit I: Calculus-I & II (Taylor/binomial series, 1st & 2nd order ODEs, Wronskian, Partial derivatives, Lagrange Multipliers).
Unit II: Vector Algebra & Differentiation (Rotations, Triple products, Gradient, Divergence, Curl, Identities).
Unit III: Vector Integration & Dirac Delta Function (Line/Surface/Volume integrals, Gauss, Green, Stokes theorems, Delta properties).
Unit IV: Orthogonal Curvilinear Coordinates (Cartesian, Spherical, Cylindrical transformations, Velocity & Acceleration analysis).

SECTION A: MULTIPLE CHOICE QUESTIONS (1 MARK EACH)

1. Which of the following functions is discontinuous at $x = 0$?

a) $\sin(x)$
b) e^x
c) $\ln(x)$
d) x^2

2. The value of $\lim_{x\to 0} (\sin x / x)$ is:

a) 0
b) 1
c) ∞
d) Undefined

3. The first three terms of the Taylor series expansion of e^x about $x = 0$ are:

a) $1 - x + x^2/2$
b) $1 + x + x^2/2$
c) $1 + x + x^2$
d) $x + x^2/2 + x^3/6$

4. The integrating factor (I.F.) for the differential equation $dy/dx + P(x)y = Q(x)$ is:

a) $\int P(x)\, dx$
b) $e^{\int P(x)\, dx}$
c) $e^{-\int P(x)\, dx}$
d) $\int e^{P(x)}\, dx$

5. If the Wronskian $W(y_1, y_2)$ of two solutions is non-zero, the solutions are:

a) Linearly dependent
b) Linearly independent
c) Equal to zero
d) Complex conjugates

6. For a second-order homogeneous differential equation with roots $m_1 = m_2 = 3$, the general solution is:

a) $c_1e^{3x} + c_2e^{-3x}$

b) $(c_1 + c_2x)e^{3x}$

c) $c_1\cos(3x) + c_2\sin(3x)$

d) c_1e^{3x}

7. A differential expression $M(x,y)dx + N(x,y)dy = 0$ is exact if:

a) $\partial M/\partial x = \partial N/\partial y$

b) $\partial M/\partial y = \partial N/\partial x$

c) $\partial M/\partial y = -\partial N/\partial x$

d) $\partial^2 M/\partial x^2 = \partial^2 N/\partial y^2$

8. In Lagrange multipliers, to extremize $f(x,y)$ subject to $\varphi(x,y) = 0$, we construct the auxiliary function:

a) $F = f + \lambda\varphi$

b) $F = f - \lambda/\varphi$

c) $F = f \cdot \lambda\varphi$

d) $F = f + \varphi$

9. Under a standard coordinate rotation, a scalar quantity:

a) Changes its sign

b) Doubles its magnitude

c) Remains invariant

d) Transforms like a vector

10. The scalar triple product $A \cdot (B \times C)$ geometrically represents:

a) Area of a parallelogram

b) Volume of a parallelepiped

c) Work done by a force

d) Area of a triangle

11. If the dot product of two non-zero vectors is zero, the vectors are:

a) Parallel

b) Perpendicular

c) Collinear

d) Opposite in direction

12. The directional derivative of a scalar field φ is maximum along the direction of:

a) $\nabla \times A$

b) $\nabla\varphi$

c) $\nabla \cdot A$

d) r

13. The divergence of the position vector $r = xi + yj + zk$ is:

a) 0

b) 1

c) 2

d) 3

14. A vector field A is called solenoidal if:

a) $\nabla \times A = 0$

b) $\nabla \cdot A = 0$

c) $\nabla^2 A = 0$

d) $\nabla\varphi = A$

15. The value of $\nabla \times (\nabla\varphi)$ for any scalar field φ is always:

a) 1

b) r

c) 0

d) $\nabla^2\varphi$

16. The value of $\nabla \cdot (\nabla \times \boldsymbol{A})$ for any vector field $\boldsymbol{A}$ is:

a) 0

b) 1

c) $\nabla^2 \boldsymbol{A}$

d) ∞

17. Gauss' Divergence Theorem converts a:

a) Line integral to surface integral

b) Surface integral to volume integral

c) Line integral to volume integral

d) Double integral to a line integral

18. Stokes' Theorem relates which of the following?

a) Surface integral and volume integral

b) Line integral and surface integral

c) Line integral and volume integral

d) Scalar field and vector field

19. The Jacobian J for the transformation from Cartesian to Polar coordinates (r, θ) is:

a) r

b) r^2

c) $r \sin\theta$

d) 1

20. The volume element dV in spherical polar coordinates is given by:

a) $dr\, d\theta\, d\varphi$

b) $r\, dr\, d\theta\, d\varphi$

c) $r^2 \sin\theta\, dr\, d\theta\, d\varphi$

d) $r \sin\theta\, dr\, d\theta\, d\varphi$

21. The value of the integral $\int_{-\infty}^{\infty} \delta(x - 3)\, dx$ is:

a) 0

b) 3

c) 1

d) ∞

22. The Dirac Delta function $\delta(x)$ is an:

a) Even function

b) Odd function

c) Periodic function

d) Imaginary function

23. The property $\delta(ax)$ is equal to:

a) $a\delta(x)$

b) $(1/a)\delta(x)$

c) $(1/|a|)\delta(x)$

d) $\delta(x)$

24. In an orthogonal curvilinear coordinate system, the scale factors h_1, h_2, h_3 for Cartesian coordinates are:

a) 1, 1, 1

b) 1, r, 1

c) 1, ρ, ρ

d) 1, r, r sinθ

25. For cylindrical coordinates (ρ, φ, z), the scale factor h_2 is:

a) 1

b) ρ

c) z

d) $\rho \sin\varphi$

26. The value of $x\,\delta(x)$ is:

a) 1 | b) x
c) 0 | d) $\delta(x)$

27. Green's theorem in a plane is a special case of:

a) Gauss' Divergence Theorem | b) Stokes' Theorem
c) Taylor's Theorem | d) Uniqueness Theorem

28. The acceleration component along θ in plane polar coordinates contains which correction term?

a) Coriolis acceleration ($2\dot{r}\dot{\theta}$) | b) Centrifugal acceleration ($r\dot{\theta}^2$)
c) Linear acceleration ($\ddot{r}$) | d) None of these

29. If $\nabla \times A = 0$, the vector field A is said to be:

a) Solenoidal | b) Irrotational
c) Divergent | d) Non-conservative

30. The value of $\int_1^5 x^2\, \delta(x - 2)\, dx$ is:

a) 0 | b) 2
c) 4 | d) 25

SECTION B: ONE-WORD / FILL IN THE BLANKS QUESTIONS (1 MARK EACH)

1. The approximation formula $(1+x)^n \approx 1 + nx$ is valid only when $|x|$ is much ________ than 1.

2. The Wronskian of the functions $\sin x$ and $\cos x$ is equal to ________.

3. An integrating factor for the differential equation $dy/dx - (1/x)y = x$ is ________.

4. The theorem that guarantees a unique solution to an initial value problem under continuity conditions is called the ________ Theorem.

5. If $z = f(x, y)$, then $dz = (\partial f/\partial x)dx + (\partial f/\partial y)dy$ is called the ________ differential.

6. The method used to find local extrema subject to equality constraints is called the Method of Lagrange ________.

7. A quantity that remains unchanged under a coordinate transformation or rotation is called an ________.

8. The vector product of two parallel vectors is always a ________ vector.

9. The magnitude of the vector product $A \times B$ represents the area of a ________.

10. The operator ∇^2 is widely known as the ________ operator.

11. The physical interpretation of the divergence of a fluid velocity field is the rate of volumetric ________ or contraction per unit volume.

12. If $\nabla\varphi$ is the gradient of a scalar field, then the line integral $\int_A^B \nabla\varphi \cdot d\mathbf{r}$ depends only on the ________.

13. The surface integral of the curl of a vector field over an open surface equals the line integral of the field around the ________ of that surface.

14. The Jacobian for the transformation from Cartesian to Spherical polar coordinates is given by ________.

15. The value of the integral $\int_{-5}^{5} (t^3 + 2)\, \delta(t)\, dt$ is ________.

16. The limit of a Gaussian function as its variance $\sigma \to 0$ represents the ________ function.

17. In an orthogonal curvilinear coordinate system, the basis unit vectors are mutually ________.

18. For spherical polar coordinates (r, θ, φ), the scale factor h_3 corresponding to φ is ________.

19. Complete the identity: $\nabla \cdot (A \times B) = B \cdot (\nabla \times A) -$ ________.

20. The line element $d\mathbf{r}$ in cylindrical coordinates is written as $d\rho\, \hat{\rho} +$ ________ $+ dz\, \hat{z}$.

21. A vector field whose curl is zero everywhere is derived from the gradient of a ________ field.

22. The total flux of a vector field out of a closed surface equals the volume integral of its ________ over the enclosed volume.

23. The value of $\int_{-\infty}^{\infty} \delta'(x)\, f(x)\, dx$ is equal to ________.

24. In curvilinear coordinates, if all scale factors are equal to 1, the coordinate system is simply ________.

25. The transformation matrix for a proper rotation in 2D has a determinant equal to ________.

26. The value of $\delta(x - a)$ for all $x \neq a$ is exactly ________.

27. The Particular Integral of $(D^2 - 4)y = e^{2x}$ cannot be found by direct substitution because it makes the denominator ________.

28. The transverse velocity component in plane polar coordinates is given by ________.

29. The number of degrees of freedom in a system constrained by 2 independent equations with 5 variables is ________.

30. The value of $\nabla \cdot \mathbf{r}$ in a 2-dimensional Cartesian plane is ________.

SECTION C: VERY SHORT ANSWER TYPE QUESTIONS (1.5 MARKS EACH)

1. State the condition for a function $f(x)$ to be continuous at a point $x = c$.

2. Write down the Taylor series expansion of $\ln(1+x)$ up to the first three non-zero terms.

3. Define an exact differential equation with an appropriate condition.

4. What is the geometrical meaning of the Wronskian of two functions?

5. State the Uniqueness Theorem for a first-order Initial Value Problem (IVP).

6. Find the integrating factor for the differential equation: $x(dy/dx) + 2y = x^3$.

7. Distinguish briefly between an exact and an inexact differential.

8. Express the total derivative of $u = f(x, y, z)$ where x, y, z are functions of a single parameter t.

9. Write down the mathematical formulation of the Lagrange multiplier method for a function of three variables with one constraint.

10. Prove that the scalar product of two vectors is invariant under a 2D rotation.

11. Show that $A \cdot (A \times B) = 0$ using standard vector properties.

12. Define a scalar field and give one physical example from classical mechanics.

13. Define a vector field and give one physical example from electromagnetism.

14. What is the physical meaning of the directional derivative of a scalar field?

15. Calculate the gradient of the scalar field $\varphi = x^2 y + z$.

16. Find the divergence of the vector field $A = x^2 i + y^2 j + z^2 k$.

17. Show that the vector field $A = yzi + zx\hat{j} + xy\hat{k}$ is irrotational.

18. Define the Jacobian matrix for a two-dimensional coordinate transformation.

19. State Green's theorem in a plane.

20. Give the physical definition of the flux of a vector field through a surface.

21. Evaluate the integral $\int_{-2}^{2} (3x^2 - 4x + 5)\, \delta(x-1)\, dx$.

22. Sketch or describe analytically the graph of a Dirac delta function.

23. Prove the property: $\delta -x) = \delta(x)$ (Even parity property).

24. Write down the property of the Dirac delta function involving a functional argument: $\delta(g(x))$.

25. Define an orthogonal curvilinear coordinate system.

26. Write down the expressions for scale factors (h_1, h_2, h_3) in cylindrical polar coordinates.

27. Write down the expressions for scale factors (h_1, h_2, h_3) in spherical polar coordinates.

28. Express the gradient operator ∇ in terms of general scale factors and unit vectors.

29. Write down the explicit expression for velocity in cylindrical coordinates.

30. State why the basis unit vectors in curvilinear systems are generally not constant vectors.

SECTION D: SHORT ANSWER TYPE QUESTIONS (2.5 MARKS EACH)

1. Discuss the continuity and differentiability of the function $f(x) = |x|$ at $x = 0$.

2. Using Taylor's series, approximate the value of $\sqrt{1.1}$ up to the second-order term.

3. Solve the first-order linear differential equation: $dy/dx + y \tan x = \sec x$.

4. Check if the functions $y_1 = e^x$ and $y_2 = e^{2x}$ are linearly independent via their Wronskian.

5. Find the general solution of the second-order differential equation: $d^2y/dx^2 - 5dy/dx + 6y = 0$.

6. Find the particular integral (P.I.) of the differential equation: $d^2y/dx^2 + 4y = \sin(x)$.

7. Show that $dF = (2xy + z^3)dx + x^2dy + 3xz^2dz$ is an exact differential.

8. Find the maximum value of $f(x, y) = xy$ subject to $x + y = 2$ using Lagrange multipliers.

9. Show that the magnitude of a vector $A = xi + yj$ remains invariant under a 2D rotation of axes.

10. Prove using vector algebra that the diagonals of a rhombus cross each other at right angles.

11. Find the unit normal vector to the sphere $x^2 + y^2 + z^2 = 3$ at the point $(1, 1, 1)$.

12. Prove the vector identity: $\nabla \cdot (\varphi A) = \varphi (\nabla \cdot A) + A \cdot (\nabla\varphi)$.

13. If $r = |r|$, prove that $\nabla(r^n) = n\, r^{n-2}\, r$.

14. Show analytically that $\nabla \times r = 0$, where r is the standard position vector.

15. Evaluate the line integral $\int_C A \cdot dr$ where $A = yi + xj$ along the line from $(0,0)$ to $(1,1)$.

16. Find the Jacobian J for the transformation from Cartesian coordinates to cylindrical coordinates.

17. Use Gauss' Divergence Theorem to evaluate ***&iint;***$_S$ $r \cdot dS$ over a closed surface enclosing volume V.

18. State Stokes' theorem and describe its physical meaning using a velocity field fluid example.

19. Show how the Dirac delta function can be represented as the limit of a sequence of rectangular functions.

20. Prove that $\int_{-\infty}^{\infty} f(x)\, \delta'(x-a)\, dx = -f'(a)$.

21. Prove that $\delta(x^2 - a^2) = (1 / 2|a|) [\delta(x-a) + \delta(x+a)]$ for $a \neq 0$.

22. Derive expressions for the line element dr and volume element dV in orthogonal curvilinear coordinates.

23. Express the Laplacian operator ∇^2 in general orthogonal curvilinear coordinates.

24. Find the components of velocity in a spherical polar coordinate system.

25. Find the components of acceleration in a cylindrical coordinate system.

26. Compare velocity expressions between cylindrical and spherical coordinate systems.

27. Show that the transformation matrix for a 2D rotation satisfies $R R^T = I$.

28. Find the work done by a force field $F = 3xyi - y^2j$ along the curve $y = 2x^2$ from $(0,0)$ to $(1,2)$.

29. Find the particular integral (P.I.) for the differential equation $(D^2 - 2D + 1)y = e^x$.

30. Show that the divergence of an inverse-square field $E = \hat{r}/r^2$ is zero everywhere except at the origin.

SECTION E: LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. Explain the concepts of continuous and differentiable functions. Illustrate with structural sketches how a curve can be continuous but not differentiable at a given point, using $f(x) = |x-1|$ as an example.

2. State and prove the method of finding the integrating factor for a non-exact first-order differential equation $M\,dx + N\,dy = 0$ when $(1/N)(\partial M/\partial y - \partial N/\partial x)$ is a function of x alone.

3. Discuss thoroughly the general solution of a second-order linear homogeneous ordinary differential equation with constant coefficients. Analyze the three distinct cases based on the discriminant of the characteristic equation.

4. Define the Wronskian. State and prove Abel's identity for the Wronskian of a second-order linear differential equation, and explain its physical and mathematical role in establishing solution independence.

5. Solve completely the regular initial value problem (IVP): $d^2y/dx^2 - 4dy/dx + 4y = 8x^2 + e^{2x}$ given initial conditions $y(0) = 1$ and $y'(0) = 0$.

6. Derive the method of Lagrange Multipliers for optimizing a function $f(x, y, z)$ subject to $\varphi(x, y, z) = 0$. Apply it to find the dimensions of an open rectangular box of maximum volume for a fixed surface area A.

7. Define a scalar product and a vector product. Discuss how vectors and their products transform under a spatial rotation of the coordinate axes. Prove that the scalar triple product is invariant under such transformations.

8. Set up the geometric interpretations of the scalar triple product and the vector triple product. Prove the identity: $A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$.

9. Define Gradient, Divergence, and Curl. Give their detailed geometrical and physical interpretations with appropriate real-world physics examples (e.g., heat flow, fluid flow, Maxwell's equations).

10. Prove the following vector identities analytically: (a) $\nabla \times (\nabla\varphi) = 0$, (b) $\nabla \cdot (\nabla \times A) = 0$, (c) $\nabla \times (A \times B) = (B \cdot \nabla)A - B(\nabla \cdot A) - (A \cdot \nabla)B + A(\nabla \cdot B)$.

11. State and prove Gauss' Divergence Theorem. Apply it to evaluate the surface integral ***&iint;*** $_S\ F \cdot dS$, where $F = 4xzi - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

12. State and prove Stokes' Theorem. Use it to evaluate $\oint_C F \cdot dr$ where $F = (2x-y)i - yz^2j - y^2zk$ and C is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by the xy-plane.

13. Define the Jacobian. Develop the transformation equations from Cartesian coordinates to Spherical Polar coordinates, and derive the corresponding Jacobian matrix and its determinant.

14. What is the Dirac Delta function? Discuss its analytical representation as the limiting case of a Gaussian function and a rectangular pulse function. Verify normalization to unity in both cases.

15. State and prove four fundamental properties of the Dirac delta function, including the scaling property $\delta(ax) = (1/|a|)\delta(x)$ and the shifting property $\int f(x)\delta(x-a)dx = f(a)$.

16. Define Orthogonal Curvilinear Coordinates. Derive general mathematical expressions for the Gradient of a scalar field and the Divergence of a vector field in terms of general scale factors h_i.

17. Derive the explicit expressions for Gradient, Divergence, Curl, and the Laplacian operator (∇^2) in the Cylindrical coordinate system.

18. Derive the explicit expressions for Gradient, Divergence, Curl, and the Laplacian operator (∇^2) in the Spherical Polar coordinate system.

19. Set up the kinematic framework for tracking moving particles. Derive the complete expressions for velocity and acceleration vectors in the Cylindrical polar coordinate system.

20. Derive the complete expressions for velocity and acceleration vectors in the Spherical polar coordinate system. Identify and physically explain terms like centripetal and Coriolis acceleration.

21. Find the general solution of the following differential equation using the method of variation of parameters: $d^2y/dx^2 + y = \sec x$.

22. Let a particle move along a path specified by $r = a(1 + \cos\theta)$ with constant angular velocity $\dot{\theta} = \omega$. Determine its radial and transverse components of velocity and acceleration as functions of θ.

23. Verify Green's theorem in a plane for the line integral $\oint_C (xy + y^2)dx + x^2dy$, where C is the closed curve bounded by the line $y=x$ and the parabola $y=x^2$.

24. Explain the concept of exact and inexact differentials in classical thermodynamics. Show that the work done $đW = P\,dV$ is an inexact differential, whereas the internal energy change dU is an exact differential.

25. Evaluate the volume integral of the function $f(x,y,z) = z$ over the region bounded above by the sphere $x^2 + y^2 + z^2 = a^2$ and below by the cone $z = \sqrt{(x^2 + y^2)}$ using spherical polar coordinates.

26. Give a rigorous mathematical treatment of the 1D Dirac delta function derivative $\boldsymbol{\delta'(x)}$. Evaluate the definite integral: $\int_{-\infty}^{\infty} \boldsymbol{e^{-x^2}\, \delta''(x)\, dx}$.

27. Show that in an orthogonal curvilinear coordinate system, the unit vectors satisfy dynamic derivative identities. Use this framework to calculate derivatives of unit vectors with respect to metrics in spherical systems.

28. A wire is bent in the form of an ellipse $\boldsymbol{x^2/a^2 + y^2/b^2 = 1}$. Find the points on the wire that are closest to and farthest from the origin using the method of Lagrange Multipliers.

29. Find the total flux of the vector field $\boldsymbol{F = x^3 i + y^3 j + z^3 k}$ passing through the entire surface of a sphere of radius $\boldsymbol{R}$ centered at the origin, checking your answer via direct surface integration and Gauss' theorem.

30. Establish the algebraic relation between the Cartesian basis unit vectors and the spherical polar unit vectors. Express this relationship as a transformation matrix and prove that it is orthogonal.

COMPREHENSIVE QUESTION BANK

Paper 2 Core II: Mechanics

Course Learning Outcomes

- To Learn the basic concepts of Rigid body dynamics, Radius of Gyration, Moment of Inertia, Non-Inertial Systems.
- To Understand the concept of Elasticity, Fluid motion and Types of Vibration.
- To understand the concept of Newtonian theory through Gravitation, Central force motion, Kepler's laws, GPS.
- To learn the concept of Special theory of Relativity, Michelson-Morley experiment, Lorentz transformation, Relativistic Doppler effect.
- Apply the basic concepts of Mechanics in laboratory experiments.

SECTION A: MULTIPLE CHOICE QUESTIONS (30 QUESTIONS • 1 MARK EACH)

Unit I: Rotational Dynamics & Non-Inertial Systems

1. The position of the center of mass of a system depends on:

a) Only the masses of the particles
b) Only the relative position of the particles
c) Both the masses and relative positions
d) The choice of the coordinate origin

2. If a coordinate system is rotating with an angular velocity ω relative to an inertial frame, the Coriolis force acting on a particle of mass m moving with velocity v' in the rotating frame is:

a) $-2m(\omega \times v')$
b) $-m\omega \times (\omega \times r)$
c) $2m(\omega \times v')$
d) $-m(\omega \times v')$

3. The moment of inertia of a solid sphere of mass M and radius R about its tangent is:

a) $2/5\ MR^2$
b) $7/5\ MR^2$
c) $3/5\ MR^2$
d) $2/3\ MR^2$

4. A particle moves in a straight line in an inertial frame. When observed from a uniformly rotating frame, its trajectory will generally appear:

a) Rectilinear
b) Parabolic
c) Curved due to Coriolis & centrifugal forces
d) Stationary

5. According to the perpendicular axis theorem, if I_x and I_y are moments of inertia of a laminar body about two perpendicular axes in its plane, its moment of inertia I_z about an axis perpendicular to its plane passing through the intersection is:

a) $I_x - I_y$
b) $I_x + I_y$
c) $\sqrt{(I_x^2 + I_y^2)}$
d) $(I_x I_y) / (I_x + I_y)$

6. The total kinetic energy of a cylinder of mass *M* and radius *R* rolling without slipping on a horizontal surface with velocity *v* is:

a) $1/2\ Mv^2$
b) $1/4\ Mv^2$
c) $3/4\ Mv^2$
d) Mv^2

7. Routh's rule states that *I = M × (Sum of squares of semi-axes / n)*. For a circular or cylindrical body, *n* is:

a) 3
b) 4
c) 5
d) 6

8. Cyclones in the Northern Hemisphere rotate in a counter-clockwise direction primarily due to:

a) Centrifugal force
b) Coriolis force
c) Gravitational pull
d) Euler forces

Unit II: Oscillations, Elasticity & Fluid Motion

9. For a damped harmonic oscillator, critical damping occurs when the damping constant *b* and natural parameters satisfy:

a) $b^2 > 4mk$
b) $b^2 = 4mk$
c) $b^2 < 4mk$
d) $b = 0$

10. At resonance, the phase difference between the displacement of a forced oscillator and the driving force is:

a) 0
b) $\pi/4$
c) $\pi/2$
d) π

11. The Quality factor (*Q*) of a weakly damped oscillator is defined as *2π* times the ratio of:

a) Avg energy stored to energy dissipated/cycle
b) Max to min displacement
c) Resonance frequency to bandwidth
d) Both a and c

12. The relation between Young's modulus (*Y*), bulk modulus (*K*), and modulus of rigidity (*η*) is given by:

a) $9/Y = 3/\eta + 1/K$
b) $9/Y = 1/\eta + 3/K$
c) $Y = 3K(1 - 2\sigma)$
d) Both a and c

13. The twisting torque per unit twist (*θ=1*) for a solid cylinder of length *L* and radius *R* is:

a) $\pi\eta R^4 / 2L$
b) $\pi\eta R^4 / L$
c) $\pi\eta R^2 / 2L$
d) $2\pi\eta R^4 / L$

14. Flexural rigidity of a beam is mathematically represented as:

a) YAk^2
b) YI_g / L
c) YI_g
d) $\eta I_g / L$

15. According to Poiseuille's equation, the volume rate of flow (*V*) depends on the radius (*r*) of the capillary tube as:

a) $V \propto r$
b) $V \propto r^2$
c) $V \propto r^4$
d) $V \propto 1/r^4$

16. Capillary ripples are fluid surface waves whose motion is primarily governed by:

a) Gravity
b) Viscosity
c) Surface tension
d) Elasticity

Unit III: Gravitation and Central Force Motion

17. The gravitational field intensity inside a uniform thin spherical shell of radius *R* at a distance *r* < *R* is:

a) $-GM/r^2$
b) $-GM/R^2$
c) Zero
d) $-GMr/R^3$

18. Under a central force field, which of the following quantities is always conserved?

a) Linear momentum
b) Kinetic energy
c) Angular momentum
d) Potential energy

19. The areal velocity of a planet revolving around the sun remains constant due to conservation of:

a) Linear momentum
b) Energy
c) Angular momentum
d) Mass

20. If a central force depends on distance as $F(r) \propto r^n$, then for the gravitational inverse-square law, *n* equals:

a) 2
b) −2
c) −1
d) 1

21. A satellite is orbiting very close to the Earth's surface. Its orbital velocity is approximately:

a) 11.2 km/s
b) 7.9 km/s
c) 5.6 km/s
d) 1.12 km/s

22. The orbit of a geosynchronous satellite must lie precisely:

a) Over the poles
b) In the equatorial plane of Earth
c) At any 12-hour inclination
d) Inside the ionosphere

23. GPS requires a minimum configuration of how many visible satellites to resolve a precise 3D position and clock bias?

a) 2
b) 3
c) 4
d) 5

Unit IV: Special Theory of Relativity

24. The null outcome of the Michelson-Morley experiment empirically disproved the existence of:

a) Photons
b) The luminiferous ether frame
c) Time dilation
d) Absolute speed of light

25. A clock moves at a speed of *$v = 0.6c$* relative to a lab. Compared to a lab clock, the moving clock runs slow by a factor of:

a) 0.8 b) 1.25
c) 0.64 d) 0.6

26. If two events occur simultaneously at different locations in one frame, then in another moving frame:

a) They must remain simultaneous b) They will not be simultaneous
c) The temporal order depends on direction d) Both b and c

27. The relativistic addition of two velocities, each equal to *$0.5c$* along the same direction, yields:

a) 1.0c b) 0.8c
c) 0.75c d) 0.95c

28. The rest mass of a photon is:

a) E/c^2 b) $h\nu/c^2$
c) Zero d) Infinite

29. The correct relativistic relation between total energy *E*, momentum *p*, and rest mass *m_0* is:

a) $E = pc + m_0c^2$ b) $E^2 = p^2c^2 + m_0^2c^4$
c) $E^2 = p^2c^2 - m_0^2c^4$ d) $E = p^2 / 2m_0$

30. Due to the relativistic Doppler effect, if a light source approaches an observer, the observed frequency will appear:

a) Redshifted b) Blueshifted
c) Unchanged d) Diminished

SECTION B: ONE-WORD / FILL-IN-THE-BLANKS (30 QUESTIONS • 1 MARK EACH)

Unit I

1. The reference frame in which the total linear momentum of a system of particles is identically zero is called the ___________ frame.

2. The quantitative value of the torque required to produce a unit angular acceleration in a body is called its ___________.

3. The parallel axis theorem requires that one of the two parallel axes must pass through the ___________ of the body.

4. Name the imaginary force that must be applied to a particle when analyzed from a non-inertial frame: ___________.

5. The deflection of a horizontally moving projectile due to the Earth's rotation is caused by the ___________ force.

6. The ratio of the angular momentum to the angular velocity vector for a symmetric body rotating about a fixed axis is defined as ___________.

7. State whether the centrifugal force is a real interaction force or a fictitious force: ___________.

8. For a rigid body rolling without slipping down an inclined plane, its acceleration is always ____________ (less than / greater than / equal to) $g\ sin\theta$.

Unit II

9. In a damped oscillator, when the system returns to equilibrium as quickly as possible without oscillating, it is ____________ damped.

10. The phenomenon of a sharp increase in amplitude when driving frequency matches natural frequency is known as ____________.

11. The theoretical upper limit for Poisson's ratio (σ) for an isotropic elastic solid is ____________.

12. A beam supported rigidly at one end and loaded at the other free end is structurally called a ____________.

13. The pendulum designed to determine g accurately by using interchangeable centers of suspension and oscillation is ____________.

14. The internal restoring torque set up per unit twist in a wire or cylindrical rod is called its ____________.

15. In Poiseuille's fluid flow model, the velocity of the liquid layer in direct contact with the tube walls is ____________.

16. Short-wavelength surface waves on a liquid where surface tension dominates over gravity are physically termed ____________.

Unit III

17. The gravitational potential inside a uniform solid sphere at its exact geometric center is equal to ____________ times the potential at its surface.

18. The trajectory of a particle moving under the action of an attractive inverse-square central force field must be a ____________ section.

19. Kepler's Third Law states that the square of the orbital period of a planet is directly proportional to the ____________ of the semi-major axis.

20. The effective mass of an isolated two-body system reduced to an equivalent one-body problem is called the ____________ mass.

21. The orbital period of a geosynchronous satellite is exactly equal to one ____________ day.

22. An astronaut feels a sensation of ____________ inside an unpowered orbiting spacecraft because both are in a state of continuous free fall.

23. The operation of GPS requires synchronization that must account for both special and general ____________ corrections.

Unit IV

24. A fundamental postulate of Special Relativity states that the speed of light in a vacuum is ____________ of the motion of the source or observer.

25. The coordinate transformations that replaced the classical Galilean transformations at near-light speeds are called the ____________ transformations.

26. The apparent shortening of the length of an object measured by an observer moving at a relativistic speed parallel to it is called ____________.

27. The invariant quantity in a Lorentz transformation involving space and time coordinates is the spacetime ____________.

28. As the velocity of a massive particle approaches the speed of light (c), its relativistic momentum approaches ____________.

29. The relativistic shift in light frequency observed when a source moves perpendicular to the line of sight is called the ____________ Doppler effect.

30. The expression representing the total energy of a particle at complete rest is given by E = ____________.

SECTION C: VERY SHORT ANSWER TYPE QUESTIONS (30 QUESTIONS • 1.5 MARKS EACH)

Unit I

1. Define Center of Mass (CoM) and write its mathematical position vector expression for a discrete multi-particle system.

2. Distinguish briefly between the Laboratory (Lab) frame and the Center of Mass (CoM) frame of reference.

3. State the principle of conservation of angular momentum. Under what structural condition is it valid?

4. Define Radius of Gyration (k) and state its physical significance in rotational dynamics.

5. State the Parallel Axis Theorem for the moment of inertia.

6. State the Perpendicular Axis Theorem for a two-dimensional plate-like structure.

7. What is a fictitious (or pseudo) force? Give two real-world examples where it is experienced.

8. Define Coriolis acceleration and write its vector formula.

Unit II

9. Write down the differential equation of motion for a damped harmonic oscillator and define the physical meaning of each term.

10. What is meant by the "sharpness of resonance"? How is it related to the damping present in the system?

11. Define Quality Factor (Q) in terms of resonance bandwidth.

12. Define Flexural Rigidity of a beam and state its SI units.

13. What is a Cantilever? Sketch or describe a single cantilever under a load at its free end.

14. Distinguish between gravity waves and ripples on a liquid surface based on their primary restoring factors.

15. State the main corrections applied to Poiseuille's equation for the flow of a liquid through a capillary tube.

16. Define the term 'Torque per unit twist' of a cylindrical wire.

Unit III

17. State Newton's Law of Gravitation in its complete vector form.

18. Distinguish between Inertial Mass and Gravitational Mass. Are they fundamentally different?

19. Define Gravitational Potential Energy. What does a negative value physically signify?

20. What is a Central Force? Provide two unique properties that characterize any central force field.

21. Define reduced mass (μ) for a two-body system and write its mathematical equation.

22. State Kepler's First Law of planetary motion.

23. What is a Geosynchronous orbit? State its approximate altitude above the Earth's surface.

24. Briefly explain the condition of weightlessness experienced by an astronaut inside an orbiting satellite.

Unit IV

25. State the two fundamental postulates of Einstein's Special Theory of Relativity.

26. What was the central experimental objective of the Michelson-Morley experiment?

27. Define Proper Length and Relative Length in relativistic mechanics.

28. What is Time Dilation? Write its mathematical formula, defining the terms used.

29. Why can a material particle with a non-zero rest mass never achieve or exceed the speed of light in a vacuum?

30. Define a "massless particle" in relativistic mechanics and provide a physical example.

SECTION D: SHORT ANSWER TYPE QUESTIONS (30 QUESTIONS • 2.5 MARKS EACH)

Unit I

1. Show that in the absence of an external torque, the total angular momentum of a system of particles remains strictly constant.

2. Establish the mathematical relation between the angular momentum vector ($\boldsymbol{L}$) and angular velocity vector (ω) for a rigid body rotating about a fixed axis.

3. Derive the expression for the rotational kinetic energy of a rigid body rotating about a fixed axis with an angular velocity ω.

4. Calculate the moment of inertia of a uniform solid cylinder of mass M, length L, and radius R about its central longitudinal axis.

5. Apply Routh's rule to determine the moment of inertia of a uniform solid sphere about its diameter.

6. Write down Euler's equations of motion for a rigid body rotating about a fixed point under zero external torque.

7. Derive an expression for the total kinetic energy of a homogeneous sphere rolling without slipping on a flat horizontal plane.

8. An observer stands on a carousel rotating with constant angular velocity ω. Explain clearly why a ball thrown straight outward appears to curve sideways to this observer.

Unit II

9. Solve the differential equation of a damped harmonic oscillator for the case of "overdamped motion" and describe its physical behavior graphically.

10. For a forced harmonic oscillator, derive the expression for the steady-state amplitude as a function of the driving frequency.

11. Show that for a weakly damped forced oscillator, the power dissipation is maximum at the exact resonance frequency.

12. Establish the elastic relation: $Y = 2\eta(1 + \sigma)$, where symbols have their usual meanings.

13. Derive an expression for the twisting torque per unit twist acting on a hollow cylindrical tube of inner radius r_1 and outer radius r_2.

14. Obtain an expression for the internal bending moment of a beam of rectangular cross-section bent into a circular arc of radius R.

15. Derive the expression for the depression (δ) at the free end of a light cantilever of length L loaded with a weight W at its end.

16. Outline the structural adjustments and mathematical logic used in a Kater's pendulum to completely eliminate the unknown effects of knife-edge curvature and air resistance.

Unit III

17. Derive an expression for the gravitational potential at a point located outside a thin uniform spherical shell of mass M and radius R.

18. Find the gravitational field intensity at a point inside a uniform solid sphere of mass M and radius R at a distance r from its center ($r < R$).

19. Show that a two-body problem interacting under a mutual central force can always be reduced to an equivalent one-body problem using the concept of reduced mass.

20. Derive the differential equation of the orbit of a particle moving under a central force field in polar coordinates ($u = 1/r$).

21. Prove that the angular momentum of a particle moving in a central force field is a constant of motion, and its path is strictly confined to a single plane.

22. Show that the total mechanical energy of a planet orbiting the sun in an elliptical path depends only on the semi-major axis a of its orbit.

23. Explain the basic operating principles of the Global Positioning System (GPS). How many satellite signals are theoretically needed to determine an absolute position on Earth, and why?

Unit IV

24. Explain the negative result of the Michelson-Morley experiment and discuss how it led directly to the development of the Special Theory of Relativity.

25. Derive the Lorentz transformation equations for space-time coordinates (x, y, z, t) assuming standard configuration with relative motion purely along the x-axis.

26. Show that the concept of simultaneity of two spatially separated events is relative and not absolute under Lorentz transformations.

27. Derive the relativistic length contraction formula $L = L_0\sqrt{(1 - v^2/c^2)}$ from the Lorentz coordinate transformations.

28. Derive the relativistic velocity addition theorem for a particle moving along the axis of frame motion. Show that if the particle moves at speed c in the moving frame, its speed is also c in the stationary frame.

29. Assuming the variation of mass with velocity, show that the relativistic mass m is given by $m = m_0 / \sqrt{(1 - v^2/c^2)}$.

30. Deduce the Relativistic Doppler effect formula for a light source moving directly away from an observer along the line of sight.

SECTION E: LONG ANSWER TYPE QUESTIONS (30 QUESTIONS)

Unit I

1. State and prove the parallel and perpendicular axis theorems for the moment of inertia of rigid bodies. Discuss their practical applications and geometric limitations in structural mechanics.

2. Set up the necessary calculus integrals and derive the expressions for the moment of inertia of a uniform solid sphere of mass M and radius R about: (i) its diameter, and (ii) a tangent line to its surface.

3. Define a flywheel. Derive an expression for the moment of inertia of a flywheel in a laboratory setup, clearly accounting for the work done against frictional torque at the axle bearings.

4. Set up the equations of motion for a uniform solid cylinder of mass M and radius R rolling down an inclined plane of angle θ without slipping. Find the linear acceleration and the minimum coefficient of static friction required to prevent slipping.

5. Define non-inertial frames of reference. Derive the complete transformation equation for the acceleration of a particle observed from a frame rotating with a constant angular velocity ω relative to an inertial frame. Identify and physically explain the origin of each fictitious term.

6. What is the Coriolis force? Derive its complete vector expression. Discuss in detail two large-scale geophysical consequences of the Coriolis force on the Earth's surface (such as atmospheric wind patterns and riverbank erosion).

7. Write a detailed analysis on the Center of Mass (CoM) frame vs. the Laboratory frame. Derive the transformation equations for velocities, linear momenta, and kinetic energies before and after a two-body elastic collision between these frames.

Unit II

8. Set up the differential equation of motion for a damped harmonic oscillator. Solve it completely for three distinct physical regimes: (i) Overdamped, (ii) Critically damped, and (iii) Underdamped. Plot or sketch the displacement-time curves for each case and highlight their engineering applications.

9. Derive the differential equation for a forced harmonic oscillator. Obtain the steady-state solution for its displacement. Find the explicit expressions for the resonance frequency and the maximum amplitude, and explain how the sharpness of resonance depends on the damping factor.

10. Define the Quality Factor (Q) of an oscillator. Prove that for weak damping, Q is equal to the ratio of the resonant frequency to the full width at half-maximum (FWHM) of the power resonance curve. Derive the expression for power dissipation.

11. Describe the construction and working of a Bar Pendulum. Derive the mathematical formula for its time period. Show how the value of acceleration due to gravity (g) and the radius of gyration (k) about the center of gravity can be extracted using graphical analysis.

12. Define the four elastic constants: Young's Modulus (Y), Bulk Modulus (K), Rigidity Modulus (η), and Poisson's Ratio (σ). Derive the expressions relating them: (i) $Y = 3K(1 - 2\sigma)$, (ii) $Y = 2\eta(1 + \sigma)$, and use them to prove that the theoretical limits of σ are $-1 \le \sigma \le 0.5$.

13. Show that the twisting couple required to twist one end of a solid cylinder of length L and radius R through an angle θ relative to its fixed other end is given by $C = \pi\eta R^4\theta / 2L$. Modify this expression for a hollow cylinder and explain why hollow shafts are structurally preferred in heavy industrial machinery.

14. Derive the expression for the internal bending moment of a uniform beam. Use this result to determine the mathematical expression for the depression produced at the center of a uniform beam of length L, supported horizontally at its two ends on knife-edges, and loaded symmetrically at its midpoint with a weight W.

15. State Poiseuille's assumptions for the steady, laminar flow of a viscous liquid through a narrow capillary tube. Derive Poiseuille's equation for the volume rate of flow ($V = \pi P r^4 / 8\eta L$). Discuss the necessary kinetic energy and end-effect corrections that must be applied for accurate real-world experimental measurements.

Unit III

16. Define gravitational potential and field intensity. Derive expressions for both the gravitational potential and field strength at an arbitrary point located: (i) outside, (ii) on the surface, and (iii) inside a uniform thin spherical shell of mass M and radius R. Plot these quantities as a function of distance from the center.

17. Derive expressions for the gravitational potential and gravitational field intensity at a point inside and outside a uniform solid sphere of mass M and radius R. Show that the field varies linearly with distance inside the sphere and follows an inverse-square law outside.

18. A particle moves under the action of a central force field. Show that: (i) the path of the particle must be a plane curve, and (ii) its areal velocity is strictly constant. Derive the differential equation of the orbit in the form $d^2u/d\theta^2 + u = -f(1/u) / (mh^2u^2)$.

19. State Kepler's three laws of planetary motion. Starting from Newton's laws of motion and gravitation, completely derive all three laws for a planet moving in an elliptical orbit around the sun.

20. Solve the differential equation of the orbit for a particle moving under an attractive inverse-square law force ($F = -k/r^2$). Show that the resulting path is a conic section, and derive the explicit conditions for the orbit to be a circle, ellipse, parabola, or hyperbola based on the system's total mechanical energy.

21. Write a comprehensive technical essay on Artificial Satellites. Include: (i) Derivation of orbital velocity and time period, (ii) Definition and derivation of parameters for a Geosynchronous/Geostationary orbit, and (iii) Explanations of the physics of weightlessness and how it affects orbital operations.

22. Discuss the Global Positioning System (GPS) from a mechanics standpoint. Explain how trilateration works using satellite signals. Detail why both Special Relativistic time dilation (speed of satellites) and General Relativistic gravitational redshift (altitude of satellites) must be corrected to maintain positioning accuracy.

Unit IV

23. Describe the experimental setup of the Michelson-Morley experiment. Derive the expected fringe shift expression based on the classical concept of a stationary luminiferous ether. Explain the significance of the null result and how it transformed modern physics.

24. State the fundamental postulates of the Special Theory of Relativity. Use them to derive the Lorentz transformation equations for space and time. Show that for low velocities ($v \leftarrow c$), these equations naturally reduce to the classical Galilean transformations.

25. Define the concept of length contraction in relativity. Derive the expression $L = L_0\sqrt{(1 - v^2/c^2)}$ using Lorentz transformations. Describe a thought experiment that highlights why length contraction is a real, measurable physical phenomenon rather than an optical illusion.

26. Derive the relativistic time dilation formula using a light-clock thought experiment. Discuss the famous "Twin Paradox" in detail, explaining how resolving the paradox requires identifying which frame undergoes physical acceleration.

27. Derive the relativistic velocity addition transformations for a particle moving with components (u_x, u_y, u_z) in a frame moving at velocity v along the x-axis. Show that the speed of light is invariant across all frames using this formulation.

28. Formulate the concept of relativistic momentum. Derive the expression for the relativistic variation of mass with velocity ($m = m_0 / \sqrt{(1 - v^2/c^2)}$) from a consideration of the conservation of momentum in an elastic collision observed from two different inertial frames.

29. Derive Einstein's famous mass-energy equivalence relation ($E = mc^2$). Show that the total relativistic energy E of a particle satisfies the invariant relation $E^2 = p^2c^2 + m_0^2c^4$. Discuss how this relation applies to massless particles like photons.

30. What is the Relativistic Doppler effect? Derive the mathematical expressions for: (i) the longitudinal Doppler effect (when the source approaches and recedes from the observer), and (ii) the transverse Doppler effect. Explain why the transverse Doppler effect has no classical analogue.