2024

Time :As in Programme

Full Marks : 100

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART-I

1. Answer the following Questions. 1x10

a. The trace of $y=3x^2$ is symmetrical about ____ axis.

b. The second term of Taylor series expansion for the function $(1-x)^{-2}$ is ____.

c. Write the unit vector perpendicular to $(\hat{i}+\hat{j})$ lies in same plane.

d. For any scaler function ϕ, the value of curl grad ϕ is ____.

e. The Dirac delta function $\delta(-x)$ = ____.

f. The function tan x is not continuous at x = ____.

g. Does the function $f(x)$ which is continuous at x_0 will be necessarily differentiable at x_0 ? (yes/no/can not say)

h. The value of $\oint_c \vec{r}\cdot d\vec{r}$ is ____.

(Turn Over)

i. In spherical polar co-ordinate system $\hat{r}\times\hat{\theta} =$ ____.

j. Does the vector triple product is associative ? (yes/no/ can not say)

PART-II

2. Answer the following questions. 2x9

a. Solve $\frac{dy}{dx} = 2x - 7$ where $y(2) = 0$.

b. Trace the curve $y = x \sin x$.

c. If $\vec{a}$ is a constant vector then show that $\vec{\nabla}\cdot(\vec{a}\times\vec{r}) = 0$.

d. If $x = r\cos\theta$, and $y = r\sin\theta$ then evaluate $\frac{\partial(x,y)}{\partial(r,\theta)}$.

e. Prove that $\hat{i}\times(\vec{a}\times\hat{i}) + \hat{j}\times(\vec{a}\times\hat{j}) + \hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$.

f. Show that $\vec{a} \neq \vec{c}$ although $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c}$.

g. Evaluate the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ from P(1,2,3) to Q(2,3,4).

h. Prove that $\vec{\nabla}\times(\phi\vec{A}) = \phi\vec{\nabla}\times\vec{A} + \vec{A}\times\vec{\nabla}\phi$.

i. For position vector $\vec{r}$ show that $\vec{\nabla}r^n = nr^{n-1}\hat{r}$.

PART-III

3. Answer any eight questions of the followings. 5x8

a. Solve the differential equation $(1+x^2)dy - (1+y^2)dx = 0$.

b. Using complementary function and particular Integral solve $y'' + 4y = 2\sin 2x$.

c. Verify the linear independence of the functions $e^{ax}sinbx$ and $e^{ax}cosbx$.

d. State and prove Green's theorem.

e. Prove that $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{F}\right)=\vec{\nabla}\cdot\left(\vec{\nabla}\vec{F}\right)-\nabla^2\vec{F}$.

f. Explain the existance and Uniqueness theorem.

g. Obtain the acceleration expression in circular cylindrical co-ordinate system.

h. Evaluate $\iint_R \left(x^2+y^2\right)dxdy$ where R is the region in 1st quadrant of XY plane with side 2 and one vertex at origin as a squire.

i. Prove that $\int_{-\infty}^{\infty}\delta(x-a)\delta(x-b)dx=\delta(a-b)$.

j. Show that scalar product of two vectors remains invariant under rotations.

PART-IV

Answer any four of the following questions. 8x4

4. What are Lagrange's multipliers ? Using them show that "The Maximum volume of solid inscribed in a sphere is a cube."

5. Establish the physical significance of scalar triple product. Obtain the total surface area and volume of a parallelepiped whose edges are $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$, $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$ and $\left(\hat{i}+2\hat{j}+\hat{k}\right)$.

6. State and prove Gauss divergence theorem. Using it obtain the volume of a sphere having radius r.

7. Derive the expression for Laplacian $\left(\nabla^2\right)$ in spherical polar co-ordinate system.

8. Solve the differential equation $\left(x^2 + y^2\right)dx - 2xydy = 0$ and obtain Dirac delta function as the limitation of Gaussian function.